RELIABLE P-HUB MEDIAN LOCATION PROBLEM IN URBAN TRANSPORTATION NETWORKS WITH MULTI ASSIGNMENT APPROACH

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Abstract

Today, with the development of urban area, the transportation has developed as a result in this area and the use of different instruments to reach the destination that moves from the shortest and most convenient way, in which there is no disruption, has become a necessity. The issue of hub locating is one of the scientific issues that have met some of the needs of urban transportation in recent decades. Therefore, in order to develop the literature of this paper, it has tried by combining the concepts of reliability to make the approaches of these issues toward increasing the reliability of transportation system; so that, communication paths are selected from among paths with minimal disruption. Therefore, this paper presents a mathematical model for the issue of reliable median p-hub locating in urban transportation network with multiple allocation approach. The paper's modeling with a bilateral approach seeks to optimize simultaneously maximizing the total sum of reliability routing and increasing demand centers that receive service without disruption.

Keywords: hub locating, vehicle routing, urban transportation,

1. Introduction

In the contemporary century, gathering of people in areas where public services are provided at the appropriate limit has led to the development of urbanization. This event led to the complexity of many of the affairs of service providing, so that the request for it has increased several times in the past several decades. One of the examples of this is the urban transportation system that is using decision sciences in order to cope with this volume of need. The issue of hub locating is one of the areas used in recent decades' studies to improve the urban transportation system, which has played an important role in this route. A valuable research in this area has been done by Gelareh and Nickel (2011). They proposed a four-index model for the problem of multi allocation hub locating without capacity which was capable in particular, to be used in urban transportation and developed its usage by developing it in their research.

They believe that in the literature, most of the structures of hub and spoke network are based on the point connection between hub nodes (i.e. the sub-graph of hub's surface is complete) and they can be implemented in our real life very rarely. In a city with at least to service-providing options (i.e. bus, metro, underground, metro rail and etc.), hub nodes usually from nodes in the bus line intersections or transfer by a proper closeness is one of the other express services. Kei truck express lines are the edges of the proposed hub, while the bus lines are supposed as the edges of spoke. Some of the nodes are potentially the hub node; the direct point to point connections between all the stops of express lines are rarely available (sub-network of express lines are not usually a complete graph). In addition,
because of the history of infrastructures (the possibility of historical architecture, environmental barriers, parks, etc.) having a complete network between hubs is not possible, even if it is beneficial from an economic perspective (Gelareh and Nickel, 2011).

A kind of hub and spoke structure in public transportation is shown in Figure 1. In this figure, rectangles are the hub nodes and circles are the spoke nodes. Then, allocations have the multiple allocation schemes and the hub surface is not complete.

![Figure 1: A typical structure hub and Spoke (From: Gelareh and Nickel, 2011)](image)

When a passenger or good is sent from a source and reaches a hub node, the number of links on the hub surface that passes before reaching their destination, is not limited to two hubs (the case in the classic hub locating issue). Three different factors of express lines path in the usual hub and spoke structure is distinguishable in figure 1-a by different rectangles that are marked by directional arrows in 1-b. Also, it should be specified that if it is profitable, the spoke communications between two hub nodes that are consistent with the bus lines between two stops of express lines are possible. This issue is shown in images of figure 1 by spoke line between nodes 7 and 13.

Despite the valuable content that they have said, it should be noted that it is right that the issues of hub locating look for a place for hub facilities that has the lowest cost for the involved system and given having the option of a communication path allocation, they try to minimize the cost of routing of the whole communication network. On the other hand, in many cases, these facilities are disrupted and service system is impaired. There are two basic root of disruption, first disruption of the service provider facility and second the available communication path being out of service. Hence, this paper aims to take advantage of the reliability issues to propel the approach of Gelareh and Nickel to a stable model environment in order to maximize the reliability of the selected routes.
The issue of hub locating does not date back a long time and perhaps it can be said that Goldman (1969) wrote the first article on the issue of hub locating. But O’Kelly (1987) was the first person who presented a known mathematical formula for the issue of hub. His role is significant in the development of the primary issues of hub especially in modeling. The first mathematical model presented by him was about the issue of median p-hub locating with single allocation, the usage of which was investigated in aerial passenger networks and reducing transportation cost by hub. Then, Campbell (1994) played a key role in the completion of the various hub models. His and his colleagues’ article in 1994 was one of the most important articles on various hub models; in addition to those mentioned above, Aykin and Klincewicz also had significant roles.

In recent years, this issue has attracted the attention of many researchers and scientists and much research has been done in this area. Figure (2-2) shows the distribution of presented articles on hubs issues. Number of articles during (1990-2007) is according to the statistics presented by Alumur and Kara (2007) in their research and Zanjirani Farahani et al (2013) have explored published articles in (2008-2013). Studies that have been examined in these papers are summarized in Table 1 below.

Table 1: Researches conducted in the field of hub locating

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Description of the researches conducted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  Goldman</td>
<td>Publishing the first article on hub locating issue</td>
</tr>
<tr>
<td>2  Berman et al. (1985)</td>
<td>Presenting two models to solve the one-median problem by considering median terminal as a queue system of M/G/1</td>
</tr>
<tr>
<td>3  O’Kelly (1987)</td>
<td>Presenting the first mathematical model to solve the problem of p-hub locating in aerial passenger networks</td>
</tr>
<tr>
<td>4  Hall (1989)</td>
<td>Analyzing and investigating the effects of time and night constraints in aerial good networks</td>
</tr>
<tr>
<td>5  Aykin (1990)</td>
<td>Calculating the created difference in the objective function based on allocated hub and presenting a model for optimal allocation of nodes to the hubs</td>
</tr>
<tr>
<td>6  Iyer and Ratliff (1990)</td>
<td>Presenting a model by considering time constraint to deliver a good on time</td>
</tr>
<tr>
<td>7  O’Kelly and Lao (1991)</td>
<td>Presenting a zero and one linear programming model for the problem raised by Hall (1989)</td>
</tr>
<tr>
<td>8  Kuby and Gray (1993)</td>
<td>Presenting a mixed integer model for solving a hub network by assuming that the place of hub is determined with the aim of reducing costs with respect to the place of hub and other parameters of the problem</td>
</tr>
<tr>
<td>9  Campbell (1994b)</td>
<td>Presenting the first linear integer programming model with $n^4 + n^2 + n$ variable and $n^4 + 2n^2 + n + 1$ constraint for solving the median p-hub locating problem with single allocation</td>
</tr>
<tr>
<td>10 Aykin (1995b)</td>
<td>Comparing the problem of hub locating under the condition where there is a direct relationship between non-hub nodes and conditions where direct relationship is not allowed.</td>
</tr>
<tr>
<td></td>
<td>Author(s) and Year</td>
</tr>
<tr>
<td>---</td>
<td>------------------</td>
</tr>
<tr>
<td>11</td>
<td>Jaillet et al. (1996)</td>
</tr>
<tr>
<td>12</td>
<td>Skorin-Kapov (1996)</td>
</tr>
<tr>
<td>13</td>
<td>Ernst and Krishnamoorthy (1996)</td>
</tr>
<tr>
<td>16</td>
<td>Ebery (2001)</td>
</tr>
<tr>
<td>17</td>
<td>Boland et al. (2004)</td>
</tr>
<tr>
<td>18</td>
<td>Costa (2007)</td>
</tr>
<tr>
<td>20</td>
<td>Contreras et al. (2009)</td>
</tr>
<tr>
<td>21</td>
<td>Eiselt (2009)</td>
</tr>
<tr>
<td>22</td>
<td>Gelare and Nickel (2011)</td>
</tr>
<tr>
<td>23</td>
<td>Alumur et al. (2012)</td>
</tr>
<tr>
<td>24</td>
<td>Karani and Sahraeian (2013)</td>
</tr>
<tr>
<td>25</td>
<td>Zanjirani Farahani et al. (2013)</td>
</tr>
</tbody>
</table>

By investigating the literature review, we found that there wasn’t any research on the reliability of created paths in hub networks in public transportation sector. Therefore, a model consistent with this condition and considering the reliability of each path in the form of maximizing the reliability of
created paths in urban transportation network was created. Thus, our contribution in improving this scientific area is summarized in three sections: first, providing a reliable integer programming model which considers two goals simultaneously and both have a significant importance. The first objective is to maximize the whole reliability of the network path that plays an important role in the acceptance of public transportation and the second objective is to maximize the tendency toward public transportation and thus to reduce urban pollution, traffic, etc.

The configuration of this study is similar to the best scientific papers. In Section 2, integer programming model is described. Section 3 introduces the method and approach of problem solving. In Section 4, it is tried to use Tehran city data in order to perform sensitivity analysis and targeted and applied analysis for the problem. Section 5 summarizes the conclusions and future recommendations.

2. Integer programming model for routing emergency services capable facilities

Before designing the model, the parameters and variables must be described. The parameters can be described as follows.

\[ X_{iklj} \]: if our communication path passes between the demand place of i and the demand place of j from the communication path between the k and l hubs, so that \( i \neq j, k \neq l \), then it would be equal to one; otherwise, it would be equal to zero.

\[ Z^1_{ijk} \]: if our communication path passes between the non-hub demand place of i and the demand place of j from the k hub, so that \( i \neq j, k \neq i, j \), then it would be equal to one; otherwise, it would be equal to zero.

\[ Z^2_{ijk} \]: if our communication path passes between the demand place of i and the non-hub demand place of j from the k hub, so that \( i \neq j, k \neq i, j \), then it would be equal to one; otherwise, it would be equal to zero.

\[ e_{ik} = 1 \] if the optimal path from i to j traverses (i, j) where at most one of i and j is a spoke node when \( i \neq j \), 0 otherwise.

\[ h_k = 1 \text{ if node } k \text{ is chosen to be a hub, 0 otherwise.} \]

\[ y_{kl} = 1, k < l \text{ if the hub edge } k \text{ and } l \text{ is established, 0 otherwise.} \]

\[ \alpha: \] a fixed amount in the interval of 0 to 1, which is used as economic index of exploitation from the communication between regular hub and the central hub.

\[ \gamma: \] a fixed amount in the interval of 0 to 1, which is defined as the reliability coefficient, as the percentage of hub's facilitating ability in providing a service without delay and crowded, when there is
only one common hub facility in the path. Therefore, this coefficient can be used in place of reliability coefficient obtained from hub.

We introduce \( W_{ij} \) as the flow from \( i \) to \( j \).

\( R_{ij} \): The reliability of the communication path between the two source nod of \( I \) and destination node of \( J \) and it should be considered that calculating this amount is based on the type of the relationship that is created in the network.

In this section, with respect to the above points, we will describe the integer programming model for the problem of the reliable hub locating in the urban transportation.

Objective function:

Formula (1) shows the objective function of the problem to maximize the reliability of the communication paths that is divided into four sections. The first part is the reliability of the path in which there are two distinct hubs. The second part is the reliability of the path in which there is is one hub and the source is a non-hub node. The third part is the reliability of the path in which there is one hub and the destination is a non-hub node.

\[
\text{Max} \sum_i \sum_{k \neq i} \sum_{jk} R_{ik} R_{ij}^{(1-\alpha)} R_{kj} W_{ij} X_{ij}
\]

\[
+ \sum_i \sum_{j=1}^{n} \sum_{k \neq j} R_{ik} Y_{ij}^{(1-\alpha)} R_{kj} W_{ij} Z_{jk}^1
\]

\[
+ \sum_i \sum_{j=1}^{n} \sum_{k \neq j} R_{ik} Y_{ij}^{(1-\alpha)} R_{kj} W_{ij} Z_{jk}^2
\]

\[
+ \sum_i \sum_{j=1}^{n} R_{ij} W_{ij} e_{ij}
\]

Constraints:

Constraints (2) will be established in hubby communication between nodes \( k \) and \( l \) when both nodes are hubs. Constraints (3), (4) and (5) are the constraints on the balance of the network current. Constraint (6) ensures that the sweep is established between the two hub nodes. Constraints (7) and (8) will create the conditions that the flow is allowed to move when the mediating node of \( k \) or \( l \) is surely established in those hubs. Constraints (9) and (10) are similar to the constraints (7) and (8) but in a situation where there is only one hub on the path. Constraints (11) and (12) guarantee that each node that will begin to flow, must pass hub node. Constraint (13) also checks that the inter-hub path selection in the path between source and destination \( (i \) and \( j) \) should both or one be hub, depending on \( i \) and \( j \).
\begin{align*}
y_{kj} & \leq h_j, \quad \forall k, l; k > l \\
y_{jl} & \leq h_k
\end{align*} \quad (2)

\begin{align*}
\sum_{l \neq i} X_{ilj} + \sum_{l \neq j} Z^1_{ijl} + e_{ij} &= 1 \quad \forall i, j; i \neq j \\
\sum_{l \neq i} X_{olj} + \sum_{l \neq j} Z^2_{ijl} + e_{ij} &= 1 \quad \forall i, j; i \neq j
\end{align*} \quad (3)

\begin{align*}
Z^2_{ijk} &= \sum_{l \neq k, j} X_{skj} + Z^1_{ijk} \quad \forall i, j, k; i \neq j, k \neq i, j
\end{align*} \quad (4)

\begin{align*}
X_{skj} + X_{skj} &\leq y_{kj} \quad \forall i, j, k, l; i \neq j, k > l
\end{align*} \quad (5)

\begin{align*}
\sum_{l \neq i} X_{skj} &\leq h_k \quad \forall i, j, k; k \neq j
\end{align*} \quad (6)

\begin{align*}
\sum_{k \neq l} X_{skj} &\leq h_i \quad \forall i, j, l; l \neq i
\end{align*} \quad (7)

\begin{align*}
Z^1_{ijk} &\leq 1 - h_i \quad \forall i, j \neq i, k \neq i, j
\end{align*} \quad (8)

\begin{align*}
Z^2_{ijl} &\leq 1 - h_j \quad \forall i, j \neq i, l \neq i, j
\end{align*} \quad (9)

\begin{align*}
Z^1_{ijk} + \sum_{l \neq k, j} X_{skj} &\leq h_k \quad \forall i, j \neq i, k \neq i, j
\end{align*} \quad (10)

\begin{align*}
Z^2_{ijk} + \sum_{l \neq k, j} X_{skj} &\leq h_k \quad \forall i, j \neq i, k \neq i, j
\end{align*} \quad (11)

\begin{align*}
e_{ij} + 2X_{ajl} + \sum_{l \neq i} X_{ili} + \sum_{l \neq j} X_{ljl} &\leq h_i + h_j \quad \forall i, j \neq i
\end{align*} \quad (12)

\begin{align*}
X_{skj}, e_{ij}, y_{kj}, h_k, Z^1_{ijk}, Z^2_{ijk} &\in \{0, 1\} \quad \forall i, j, k, l
\end{align*} \quad (13)
But the other thing that was done was estimating the implementation cost of each reliability plan to see how much will be added to the cost, by increasing it. To calculate the total cost, the equation (15) will be used.

\[
\text{Max} \sum_{i} \sum_{k} \sum_{l \neq i} \sum_{j \neq i} \alpha C_{ik} C_{ij} W_{ij} X_{ikj} + \sum_{i} \sum_{j \neq i} \sum_{k \neq i, j} \alpha C_{ik} C_{ij} W_{ij} Z^1_{ijk} + \sum_{i} \sum_{j \neq i} \sum_{k \neq i, j} \alpha C_{ik} C_{ij} W_{ij} Z^2_{ijk} + \sum_{i} \sum_{j \neq i} C_{ij} W_{ij} e_{ij} \]

(15)

3. Analysis of Computational results

In this section, it is tried have the model analysis for the urban services of Tehran with a functional look. Hence, 22 districts of Tehran are considered as the demand centers and district 8, which has a strategically good position and the establishment of hub is possible there, is considered as the potential place for hub facilities. It should be noted that the fair distribution of service-providing centers are also on the agenda. Hubs are considered here as a terminal. In Figure 2 we can see the map of Tehran’s districts.

Figure 2: Map of Tehran on 22 area its segregation
The amount of connections between districts is considered as a true random number between 50 and 100 and the distance between the areas was calculated with the help of Google Map website. Output data in of the objective function of the problem with respect to changes in the parameters α and \( \gamma \) is provided in Table 2.

To understand changes more clearly, four plans with the existing conditions are created in Figure 3 and output images are shown on the map in which images 4, 5, 6 and 7, are about cases 1 to 4, respectively.

Table 2: Results of the output of the objective function for different values of the parameters α and \( \gamma \)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Nodes Number</th>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>Cpu time (sec)</th>
<th>Reliability Objective function</th>
<th>Total Cost for Network</th>
<th>Nodes selected as Hub</th>
<th>Number of opening Hub</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tehran City</td>
<td>22</td>
<td>0.1</td>
<td>0.1</td>
<td>25.952</td>
<td>380.698066</td>
<td>3, 10</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>24.141</td>
<td>443.954851</td>
<td>2, 8, 9</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9</td>
<td>0.9</td>
<td>23.408</td>
<td>492.231005</td>
<td>2, 8, 9, 14, 19</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>23.657</td>
<td>466.255798</td>
<td>8, 9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>25.415</td>
<td>557.709297</td>
<td>2, 7, 9, 16</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>24.164</td>
<td>625.323021</td>
<td>3, 5, 8, 15, 17</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>23.025</td>
<td>609.33737</td>
<td>1, 20, 22</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>24.872</td>
<td>631.53753</td>
<td>2, 8, 15, 18, 21</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>24.407</td>
<td>675.365423</td>
<td>1, 5, 8, 11, 13, 18, 20, 22</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
### Figure 3: Design of experiments for study the dynamics for different values of $\alpha$ and $\gamma$

<table>
<thead>
<tr>
<th>Parameter $\gamma$</th>
<th>0.1</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter $\alpha$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$\text{Case 1}$</td>
<td>$\text{Case 2}$</td>
</tr>
<tr>
<td>0.9</td>
<td>$\text{Case 3}$</td>
<td>$\text{Case 4}$</td>
</tr>
</tbody>
</table>

### Figure 4: Tehran's Map for output routes of case 1
Figure 5: Tehran's Map for output routes of case 2

Figure 6: Tehran's Map for output routes of case 3
As can be understood from maps in figures 4, 5, 6 and 7 by increasing change and trend of $\alpha$ and $\gamma$ parameters reliability will also increase. The number of established hub facilities will also increase with the same trend. If we consider Figure 8, the bar chart of cost and level of reliability for all four cases is plotted. According to Figure 8, strategy can be easily chosen and they will be options 4 and 3, respectively.
6. Summarizing and conclusion

By presenting a model of multiple allocation reliable hub locating in the urban transportation, this paper was able to introduce a unique and valuable model by taking into account the reliability of the communication paths and considering that it has been implemented in Tehran metropolitan, it has operational value. Also by providing sensitivity analysis, it will help urban transportation decision makers to have an open look at available option in their decision making process. It should be noted that this study can be extended by using meta-heuristic solution methods to solve it in order to solve the problem in broader terms.

References